of the velocity and temperature distributions can be obtained from the profiles plotted along the symmetry line between two adjacent fins. Figures 4 and 5 present such profiles for a few typical cases. As the Rayleigh number increases, the velocity and temperature distributions tend to become more uniform, thus increasing the gradients at the base surface. Also, it is interesting to note in Fig. 5 that the closely spaced fins ($S = 0.1$) exert a strong influence throughout the inter-fin space so that the values of $w/\bar{w}$ and θ remain rather small and uniform for $Y < 1$.

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TURBULENT HEAT AND MASS TRANSFER IN NEWTONIAN AND DILUTE POLYMER SOLUTIONS FLOWING THROUGH ROUGH PIPES

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NOMENCLATURE

- d pipe diameter
- e roughness height
- f friction factor
- f_s friction factor for a smooth pipe
- Nu Nusselt number
- Pr Prandtl number
- Re Reynolds number
- Sc Schmidt number
- Sh Sherwood number
- T time period for sublayer development
- T^+ dimensionless time period, Tv_0^2/ν
- v_0 friction velocity.

Greek symbol
- ν kinematic viscosity.

INTRODUCTION

THE CONCEPT of an eddy diffusivity has been widely used to discuss the increase in heat and mass transfer owing to the surface roughness as well as the studies of transport processes near smooth surfaces [1–6]. On the other hand, the concept of a surface renewal, which has been successfully used to predict turbulent heat and mass transfer rates from smooth surfaces to flowing fluids [7], has been scarcely applied. Hughmark [8] utilized the penetration model to rough surface pipes. In Hughmark's work, the influence of surface roughness on transport processes is taken account of only in the turbulent core region. Furthermore, the constants in the expressions for the mass transfer coefficients in the laminar sublayer and the transition regions were determined by comparison with heat and mass transfer data in smooth pipes.

In this work, the applicability of the surface renewal concept is discussed. The proposed model based on the periodic transitional sublayer model (see ref. [9]) is compared with the available experimental data on turbulent heat and mass transfer in a rough pipe. The influence of polymer additives is also discussed.

SURFACE RENEWAL MODEL

It has been shown [9, 10] that the periodic transitional sublayer model proposed by Pinczewski and Sideman [9] provides a satisfactory representation of turbulent heat and mass transfer in Newtonian and non-Newtonian fluids flowing through smooth pipes. We discuss the effect of surface roughness on the transfer rates using the periodic transitional sublayer model.

When the Schmidt number is sufficiently high, the rate of mass transfer in a smooth pipe is determined by the rate of renewal of the thin wall-layer. The roughness causes disturbances in the viscous sublayer which penetrate to the wall and promote turbulent transport. Therefore, the thickness of the viscous sublayer and of the wall-layer decreases with increasing roughness. As the result, even at high Schmidt numbers, the thickness of the concentration boundary layer might be larger than that of the wall-layer. In other words, the effect of the thin wall-layer is negligible and the resistance to mass transfer is mainly due to the periodically developing sublayer flow. For $Sc > 1$, the resistance of the turbulent core is small in comparison to that within the sublayer and can be assumed to be negligible. This situation for mass transfer from a rough surface at high Schmidt numbers is corresponding to the mass transfer in a smooth pipe for $Sc = O(1)$. The validity of this assumption is verified *a postiori* by a comparison of the predictions of the model with experimental data.

An expression for the Sherwood number for Newtonian fluids and dilute polymer solutions under the maximum drag reduction condition for $Sc = O(1)$ may be written as [9, 10]

$$Sh = \frac{1}{T^+}\sqrt{\left(\frac{f}{2}\right)}\, Re\, Sc^{1/2}\,(1.11 + 0.44\, Sc^{-1/3} - 0.70\, Sc^{-1/6}). \tag{1}$$

As described above, this equation might be applicable to discuss turbulent mass transfer in a rough pipe at fairly large Schmidt numbers.

Because of the lack of pertinent information, the dimensionless time period of sublayer development T^+ for rough pipes is assumed, as a first approximation, to be consistent with that for smooth pipes. Considering the values of T^+ for Newtonian fluids ($T^+ = 13.1 \sim 18.4$) and maximum drag reducing fluids ($T^+ = 42.1 \sim 59.2$) presented in the literature [9–15], we assume that for Newtonian fluids

$$T^+ = 15.0, \tag{2}$$

and for maximum drag reducing fluids

$$T^+ = 50.0. \tag{3}$$

Substituting equation (2) into equation (1) gives

$$Sh = 0.471 f^{1/2}\, Re\, Sc^{1/2}\, (1.11 + 0.44\, Sc^{-1/3} - 0.70\, Sc^{-1/6}). \tag{4}$$

For sand-type roughness, the friction factor for Newtonian fluids is [16]

$$f^{1/2} = 0.197 (e/d)^{0.15}, \tag{5}$$

and equation (4) can be written as

$$Sh = 0.00929 (e/d)^{0.15}\, Re\, Sc^{1/2}\, (1.11 + 0.44\, Sc^{-1/3} - 0.70\, Sc^{-1/6}). \tag{6}$$

Substituting equation (3) into equation (1), we have an expression for mass transfer in a rough pipe under the condition of maximum drag reduction

$$Sh = 0.0141 f^{1/2}\, Re\, Sc^{1/2}\, (1.11 + 0.44\, Sc^{-1/3} - 0.70\, Sc^{-1/6}). \tag{7}$$

No reliable correlation of f for a maximum drag reducing fluid in rough pipes is available.

DISCUSSION

Figure 1 compares predictions from the present model, equation (6), with the available experimental data [1, 3, 4, 17] and correlations [1, 18, 19]. The experimental data for $0.0024 < e/d < 0.165$ show reasonable agreement with predictions. The present model, which agrees well with Dipprey and Sabersky's [1] correlation, lies somewhat below the correlation based on the three-zone model [18]

$$Sh = 0.0105 (e/d)^{0.15}\, Sc^{1/2}\, Re, \tag{8}$$

and somewhat above the Nunner correlation [14] for $Re > 5 \times 10^4$

$$Sh = \frac{(f/2) Re\, Sc}{1 + 1.5\, Re^{-1/8}\, Sc^{-1/6}\, [Sc(f/f_s) - 1]} \tag{9}$$

where f/f_s is the ratio of rough to smooth friction factors at the same Reynolds number.

Predictions of equation (7) are compared with results for heat transfer data under the maximum drag reduction condition obtained by Debrule and Sabersky [20] (Fig. 2). Since there is no reliable correlation for the friction factor of drag-reducing fluid flow in rough pipes, we use the experimental data for the friction factor measured by Debrule and Sabersky [20] to estimate the Sherwood numbers. It can be seen that the present model is in reasonable agreement with the data and the model based on the Levich three-zone model [18]

$$Sh = 0.0111\, Sc^{1/2}\, Re\, f^{1/2}. \tag{10}$$

Considering the models based on the assumption of a fairly high Schmidt number are adequate for $Sc > 5$ [10, 18], the present model is also expected to be applicable in the region of $Sc > 5$.

CONCLUSION

The surface renewal concept is applied to turbulent heat and mass transfer in rough pipes. The proposed model is in reasonable agreement with the available experimental data for Newtonian and dilute polymer solutions. It may therefore be concluded that the present model yields a satisfactory

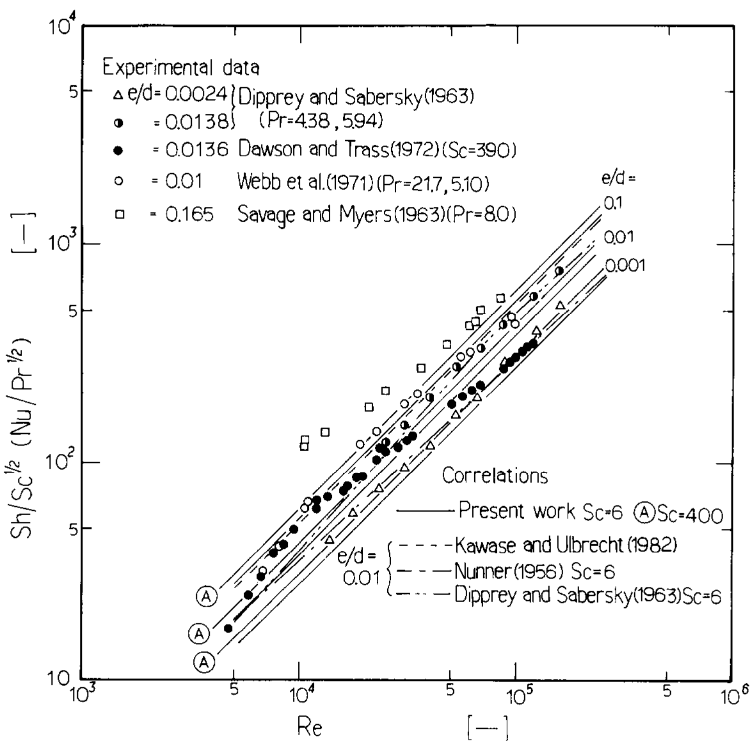

FIG. 1. Comparison of the present model with data for heat and mass transfer (Newtonian fluid).

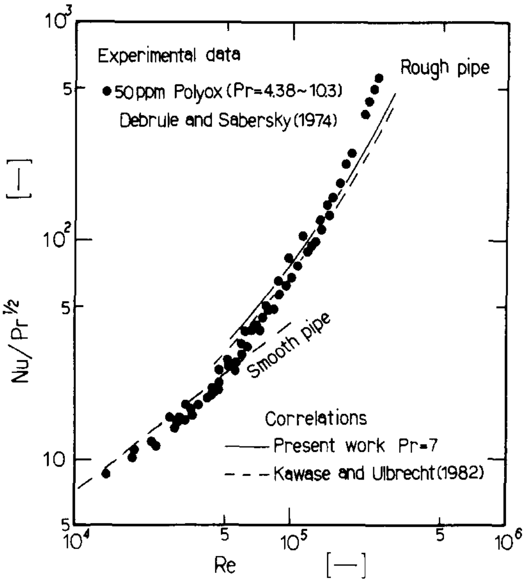

FIG. 2. Comparison of the present model with data for heat and mass transfer (dilute polymer solution under the maximum drag reduction condition).

representation of the heat and mass transfer processes in rough pipes. Unfortunately, more detailed discussion is hampered by the lack of experimental data for dilute polymer solutions and suitable parameters to describe the roughness. A more precise model based on the surface renewal concept must await a greater wealth of knowledge for the bursting phenomena in a rough pipe. Furthermore, it has been known that some of the scatter in the experimental data for dilute polymer solutions is due to degradation of the polymers and entrance effects [21, 22]. Therefore the experimental data carried out taking these effects into consideration is also required.

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THE THERMAL RESISTANCE OF A COMPOSITE HOLLOW SPHERE

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YOYANOVICH *et al.* [1] computed the thermal resistance of a hollow sphere subjected to an arbitrary polar flux. One of their aims was to determine the thermal resistance for isothermal polar regions. One of their assumed flux distributions did give an approximately uniform polar temperature distribution, at least for a small polar angle α and for thick shells. In this note the isothermal problem is solved using an analysis developed by Collins [2].

Figure 1 shows a composite spherical shell composed of shells of external and internal radii (a, b) and (b, c), and thermal